ACTIVITY 11

Hundreds Magic

Materials:

- ☒ paper
- ☐ transparency of activity master
- ☐ centimeter grid paper

Overview:

Students will be so amazed at the patterns they find in the hundreds chart they will think you are a "mathemagician". Once hooked, they will use algebra to understand the magic.

Vocabulary: variable, formula, proof

PROCEDURE

Skills:

- Multiplying monomials and binomials
- Finding patterns
- Using algebraic proofs

Good Tip!

If students are seated in groups the effectiveness of this activity is increased. They will see that their neighbors achieve the same results and patterns by circling different numbers.

1 The students will need to make a hundreds chart like the activity master. Alternately, you may wish to provide grid paper for this or simply distribute copies of the transparency master while you use the transparency on the overhead projector.

2 Ask the students to circle any four adjacent numbers which form a square. We will use 7, 8, 17, and 18 as an example. Tell them to add the two diagonals of the square and compare the results. They will notice that 7 + 18 = 8 + 17. Have them try the same process with a different set of four numbers.

3 Younger students will enjoy simply exploring the patterns in the chart without generalizing the relationships with formulas. More advanced students may be able to explain why the patterns occur, without using formal algebra. If your students are ready for the proof, this is the time to demonstrate it. Most students should be able to follow the explanation after their exploration of the chart.

Notice that for any beginning number, the next number is n + 1. The numbers below these are n + 10 and n + 11. Thus the sums of the diagonals are:

$$(n) + (n + 11) \text{ and } (n + 1) + (n + 10)$$

Combining terms gives us:

$$2n + 11 = 2n + 11.$$

4 Next, ask students to multiply diagonals and compare the results. They will see that the products are not equal. In our example, we get 7 x 18 = 126 and 8 x 17 = 136. However, when they try other locations, they will see that the second answer is *always* ten more than the first.

5 Once again, the reason can be explained fairly simply:

$n(n + 11) = n^2 + 11n$ $\qquad$ $(n + 1)(n + 10) = n^2 + 11n + 10$

Journal Prompts:

Are the *differences* of the diagonals always equal? Explain why this is or is not true.

Make up an arrangement of numbers other than the four number square. Describe any patterns or relationships that you find.

Homework:

Ask students to explore patterns found in other arrangements of numbers other than the four number square explained above. Some examples are shown in the pattern key on the following page. However, there are many more patterns and proofs for the students to discover.

Taking a Closer Look: B

Ask students to explore these same relationships and others on any calendar page. An activity master is provided for this. What similarities and differences occur?

Advanced students can incorporate practice with negative numbers using the second activity master.

Assessment:

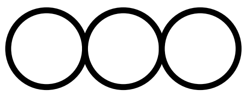
Allowing students to work in small groups will provide the opportunity for self assessment. Since all the patterns can be generalized, a single formula should result when students explore a given arrangement of numbers. Some sample patterns and proofs are offered on the following page.

Good Tip!

Ask students to find their own patterns in the chart and present them to the class.

For example, a student may wish to prove why the sum of any row is ten times the last number minus 45.

Pattern Key:

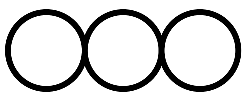


Three-in-a-Row:

Pattern 1: The sum of the three numbers equals three times the middle number.

Proof: If "n" is the center then the left number is $n - 1$, and the right number is $n + 1$. Thus their sum is:

$$(n - 1) + n + (n + 1) = 3n - 1 + 1 = 3n$$

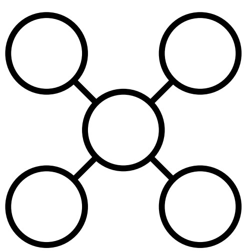


Three-in-a-Row:

Pattern 2: The product of the left and right number is one less than the square of the center..

Proof: Their product can be written:

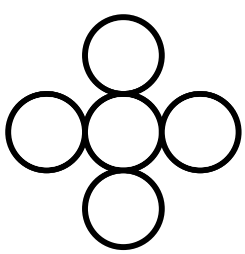
$$(n - 1)(n +1) = n^2 + n - n + 1 = n^2 - 1$$



Five-Point: The average of the four corners is equal to the center.

Proof: If "n" is the center then the corners are $n - 11$, $n - 9$, $n + 9$, and $n + 11$. Thus the average is:

$$[(n - 11) + (n - 9) + (n + 9) + (n + 11)] \div 4 = (4n) \div 4 = n$$



Cross: The product of the top and bottom number is 99 less than the product of the left and right numbers.

Proof: If "n" is the center number, then the product of the top and bottom numbers is:

$$(n - 10)(n + 10) = n^2 - 100$$

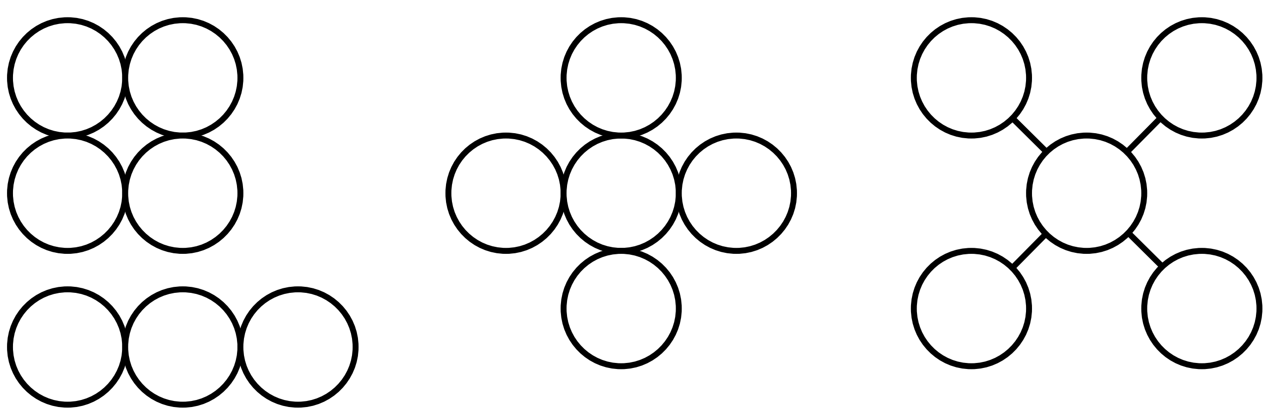
The product of the left and right numbers is:

$$(n - 1)(n + 1) = n^2 - 1$$

$$\text{and } (n^2 - 100) = (n^2 - 1) - 99$$

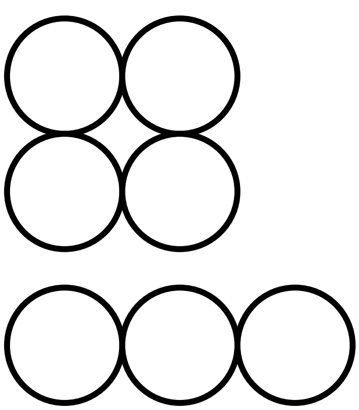
Hundreds Magic: 1 – 100

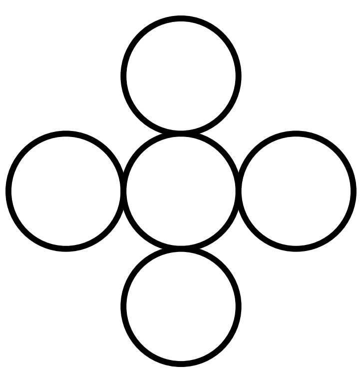
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

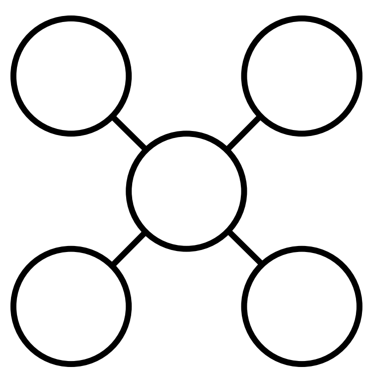


Hundreds Magic: -49 – 50

-49	-48	-47	-46	-45	-44	-43	-42	-41	-40
-39	-38	-37	-36	-35	-34	-33	-32	-31	-30
-29	-28	-27	-26	-25	-24	-23	-22	-21	-20
-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50







Hundreds Magic: Calendar

SUN	MON	TUE	WED	THU	FRI	SAT